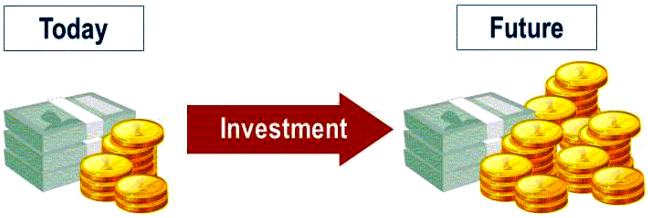
Chapter 1
Time Value of Money

The concept of time value of money

- The time value of money means that a dollar received today is worth **more** than a dollar received in the future.
- This is because a dollar received today can be **invested** and earn **interest**.
  - Money **grows** over time.
  - The additional amount reflects the **time value** of money



Compounding

- Compounding means that the profit earned on an investment is **reinvested** to make even more profit.
- This is the process of finding **future value**.
- **Future value** is the value at the end of a time period from a sum of money today.
- For example:
  - Most investors receive **compound** interest if they deposit money in banks. The **interest** earned in the current period becomes part of the **deposit** and the total sum earns **interest** in subsequent periods.

Example 1

Martin has just deposited $1,000 in a savings account at an interest rate of **5%** per annum (p.a.).

One year later: Amount = $\$1,000 \times (1 + 5\%) = \$1,050$

Two years later: Amount = $\$1,000 \times (1 + 5\%)^2 = \$1,102.5$

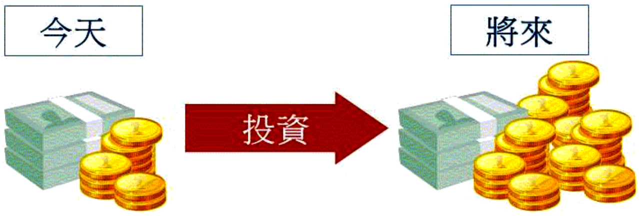
Discounting

- Discounting is the process of finding **present value**.
- **Present value** is the current value of a future sum of money.
- This is useful when we need to know how much money was **originally** needed to grow (compound) into a certain amount in the future.
- For example, How much do I need to save now so that I can have enough money to buy a new computer next year?

第 1 章
金錢的時間值

金錢的時間值的概念

- 金錢的時間值指今天收到的一元比將來收到的一元**更有**價值。
- 這是因為今天的一元可用作**投資**以賺取**利息**。
  - 金錢隨着時間而**增多**。
  - 額外多收的金額正好反映金錢的**時間值**。



複息計算

- 複息計算的意思是利用投資所得的利潤**再作投資**，從而賺取更多利潤。
- 這亦即是計算**未來值**的方法。
- **未來值**指現在的一筆錢在未來某個時段完結時的價值。
- 例如:
  - 投資者把錢存入銀行，大都可以獲取**複息**（又稱**複利**）,即今期賺取的**利息**會成為下期**存款**的一部分，一併賺取**利息**。

例子 1

浩然剛把 1,000 元存入儲蓄帳戶，年利率為 5%。

一年後: 本利和 $= \$1,000 \times (1 + 5\%) = \$1,050$

兩年後: 本利和 $= \$1,000 \times (1 + 5\%)^2 = \$1,102.5$

貼現計算

- 貼現計算是計算**現值**的方法。
- **現值**是未來的一筆錢在現時的價值。
- 有時，我們須要知道要投放多少金額，才能令數額在未來的一個期限內，複息增長至某個價值。
- 例如，現在我須要存多少錢，才可在明年有足夠金錢購買一部新電腦？

Calculating the time value of money

Future value (FV)

- Future value is the value at the **end** of a time period from a sum of money **today**.

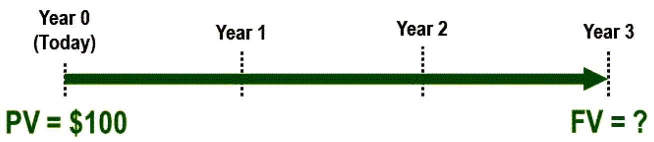
Example 2

Today, Mr Lee deposits $100 in a bank for three years.

✧ The annual interest rate is 10%.

✧ The interest is compounded annually.

How much money will Mr Lee get back after 3 years?



FV = PV x $(1 + i)^n$

Three years later, Mr Lee can get back:

FV = $100 × $(1 + 10\%)^3$ = $133.1

Example 3

Today, Ms Wong deposits $3,000 in a bank for three years.

✧ The annual interest rate is 8%.

✧ The interest is compounded annually.

How much money will Ms Wong get back after 3 years?

FV = PV x$(1 + i)^n$

FV = $3,000 × $(1 + 8\%)^3$ = $3,779.1

At the end of three years, Ms Wong will get $3,779.1.

Present value (PV)

- Present value is the **current** value of a **future** sum of money.
- It is the amount needed **today** to grow into a specific sum in the **future**.

Example 4

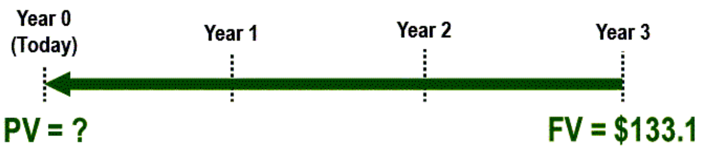
Today, Mrs Lo invests in a financial product which promises her $133.1 (principal + interest) in three years.

✧ Her cost of capital is 10%.

✧ Cost of capital is the rate of return that one can earn from the next best alternative.

✧ The investment return is compounded once a year.

How much money does Mrs Lo have to invest in order to receive the promised return?



$$PV = \frac{FV}{(1+i)^n}$$

Mrs Lo has to invest: PV = $133.1 ÷ $(1 + 10\%)^3$ = $100

計算金錢的時間值

未來值 (FV)

- 未來值指**現在**的一筆錢在未來某個時段**完結**時的價值。

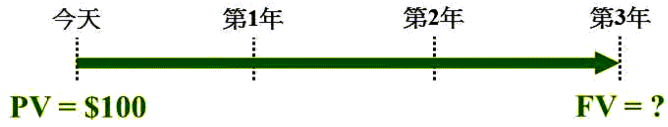
例子 2

李先生在今天把 100 元存入銀行，為期三年。

✧ 銀行的年利率為 10%。

✧ 每年以複息計算利息。

三年後，李先生可取回多少存款？



FV = PV x $(1 + i)^n$

三年後，李先生可取回:

FV = $100 × $(1 + 10\%)^3$ = $133.1

例子 3

今天，王女士把 3,000 元用作投資，為期三年。

✧ 年利率為 8%

✧ 每年以複息計算利息。

三年後，王女士可取回多少錢？

FV = PV x$(1 + i)^n$

FV = $3,000 × $(1 + 8\%)^3$ = $3,779.1

第三年完結時，王女士可取回 3,779.1 元。

現值 (PV)

- 現值是**未來**的一筆錢在**現時**的價值。
- 它可理解為：要令金錢在**未來**增長至某個價值（未來值）而**現在**所需的金額。

例子 4

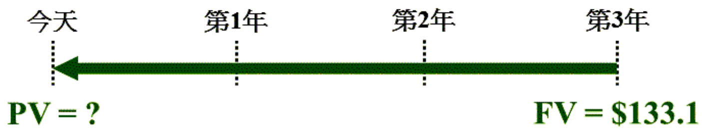
盧太太在今天投資某項金融產品；該金融產品保證她在三年後能獲得 133.1 元（本金 + 利息）。

✧ 盧太太的資本成本為 10%。

✧ 資本成本是指投資在次佳選擇的回報率。

✧ 投資回報每年以複息計算。

盧太太須投資多少金錢，才能在三年後獲得保證回報？



$$PV = \frac{FV}{(1+i)^n}$$

盧太太須投資: PV = $133.1 ÷ $(1 + 10\%)^3$ = $100

Example 5

Mr Cheung plans to buy a car, which costs $200,000, in three years.

✧ He has decided to put a lump sum in a time deposit account for **three years**

✧ at an annual interest rate of **8%**.

✧ The interest is compounded annually.

How much should Mr Cheung put in the account today?

PV = $\$200,000 \div (1 + 8\%)^3 = \$158,766.4$

Mr Cheung needs to put $158,766.4 in the time deposit account today.

Example 6

Lisa's uncle offers her two choices for her birthday gift:

Choice 1: Receiving $1,000 now
Choice 2: Receiving $1,100 one year later
Assume that the highest interest rate for a $1,000 deposit is **3%** per year, compounded annually.

Which choice is better for Lisa financially?

To choose between the two alternatives, Lisa can compare their present values:

Choice 1: PV = $1,000
Choice 2: $PV = FV \div (1 + i)^n$
$= \$1,100 \div (1 + 3\%)^1$
$= \$1,068.0$

Since the PV of Choice 2 is higher, this is a better gift.

Using net present value to make investment decisions

- Net present value (NPV) is the **PV** of future **net cash inflows**, less the **initial** outlay.
  - For example, if the present value of the future net cash inflows of an investment is $10,000 and the initial outlay is $3,000, then the NPV is **$7,000**.
- It is useful in making **investment** decisions.

例子 5

張先生計劃在三年後購買一輛價值 200,000 元的汽車。

✧ 他決定把一筆錢存入定期存款帳戶，**為期三年**。

✧ 年利率為 **8%**。

✧ 每年以複息計算利息。

張先生現在應存入多少錢？

PV = $\$200,000 \div (1 + 8\%)^3 = \$158,766.4$

張先生現在須把 158,766.4 元存入定期存款帳戶。

例子 6

今天是麗珊的生日，她從叔叔兩份生日禮物中挑選一份:

選擇（一)：現在收取 1,000 元
選擇（二)：一年後收取 1,100 元
假設 1,000 元可獲得的最高存款年利率為 3%，每年以複息計算利息。

對麗珊而言，哪項選擇在財務上較為明智？

麗珊作出選擇前，可先計算兩份禮物的現值:

選擇（一）： PV = $1,000
選擇（二）： $PV = FV \div (1 + i)^n$
$= \$1,100 \div (1 + 3\%)^1$
$= \$1,068.0$

由於選擇（二）的現值較高，麗珊應該選擇第二項。

利用淨現值作出投資決策

- 淨現值等於未來**淨現金流入量**的**現值**減去**最初**的支出。
  - 例如在某項投資中，未來淨現金流入量的現值是 10,000 元，而最初的支出是 3,000 元，淨現值就是 **7,000 元**。
- 淨現值有助人們作出**投資**決策。

Example 7

Mr Kwok is considering purchasing a new computer for his business, which will cost him **$40,000**.

✧ He expects the computer will be used for three years, and that it will be sold for **$5,000** at the end of Year 3.

✧ He estimates that the future cash inflows generated by this computer will be **$15,000** at the end of each year for the next three years.

✧ The cost of capital is **10%** per annum.

Should Mr Kwok buy the computer?

To answer the question, we need to calculate the NPV.

Step 1 Determine the cost of the computer.
$40,000

Step 2 Estimate the annual net cash inflows from using this computer for the next three years.
$15,000 each year

Step 3 Estimate the market value of the computer in Year 3.
$5,000

Step 4 Calculate the PVs of all the annual net cash inflows and the computer's market value in Year 3.

Total PV of annual net cash inflows
= ($15,000 ÷ 1.1) + ($15,000 ÷ 1.1^2) + ($15,000 ÷ 1.1^3)
= $37,302.78
PV of the computer's market value at the end of Year 3
= 5,000 ÷ 1.1^3
= $3,756.57

Step 5 Add up all the PVs (total PV), i.e., PVs of the annual net cash inflows and the computer's market value in Year 3.
Total PV = $37,302.78 + $3,756.57
= $41,059.35

Step 6 Find the NPV.
NPV = (PVs of all annual net cash inflows from using the computer + PV of the computer's market value in Year 3) – Initial cost of the computer
NPV = Total PV – Initial outlay
= $41,059.35 – $40,000
= $1,059.4

例子 7

郭先生正考慮為他的公司購買一台成本為 **40,000 元**的新電腦。

✧ 他預計該電腦可使用三年，三年後可以 **5,000 元**把它賣出。

✧ 他估計在未來三年內，該電腦在每年年底 會帶來 **15,000 元**的現金流入量。

✧ 郭先生的資本成本是每年 **10%** 。

郭先生應該購買這台電腦嗎？

我們可運用淨現值的概念來回答這個問題：

第 1 步 找出電腦的成本。
40,000 元

第 2 步 估計未來三年內，使用這台電腦每年帶來的淨現金流入量。
每年 15,000 元

第 3 步 估計電腦在第三年的市值。
5,000 元

第 4 步 計算每年淨現金流入量的現值，以及三年後電腦市值的現值。

每年淨現金流入量的總現值
= ($15,000 ÷ 1.1) + ($15,000 ÷ 1.1^2) + ($15,000 ÷ 1.1^3)
= $37,302.78
第三年年底電腦市值的現值
= 5,000 ÷ 1.1^3
= $3,756.57

第 5 步 把第 4 步的所有現值相加（總現值），包括三年後電腦市值的現值。
總現值 = $37,302.78 + $3,756.57
= $41,059.35

第 6 步 計算淨現值。
淨現值 =(使用電腦帶來的每年淨現金流入量的總現值 + 第三年電腦市值的現值) – 電腦最初的成本
淨現值 = 總現值 – 最初的支出
= $41,059.35 – $40,000
= $1,059.4

Step 7 (i) If the NPV is positive, Mr Kwok should buy the computer as the financial benefit in doing so outweighs its cost.
(ii) If the NPV is negative, Mr Kwok should not buy the computer as he will suffer a loss.
(iii) If the NPV is equal to zero, Mr Kwok should buy the computer as he will earn exactly the cost of capital from the investment.
As the NPV is **positive**, Mr Kwok should purchase the computer

Example 8

Mr Wong wants to purchase a dishwasher for his restaurant. He is considering *Brand A* and *Brand B*:

✧ ***Brand A*** costs $10,000 and ***Brand B*** costs $12,000.

✧ Both of them can be used for three years and can be sold for $1,000 at the end of Year 3.

✧ Estimated net cash inflows generated at the end of the next three years:

	Year 1	Year 2	Year 3
Brand A	$5,000	$4,000	$3,000
Brand B	$6,000	$5,000	$4,000

If the cost of Mr Wong's capital is 8% per annum, which brand should he buy?

Compare the NPVs of the two dishwashers:

Brand A

Total PV of annual net cash inflows
= ($5,000 ÷ 1.08) + ($4,000 ÷ 1.08^2) + ($3,000 ÷ 1.08^3)
= $10,440.48

PV of its market value at the end of Year 3
= $1,000 ÷ 1.08^3 = $793.83

Total PV = $10,440.48 + $793.83 = $11,234.31

NPV = $11,234.31 – $10,000 = $1,234.3

Brand B

Total PV of annual net cash inflows
= ($6,000 ÷ 1.08) + ($5,000 ÷ 1.08^2) + ($4,000 ÷ 1.08^3)
= $13,017.58

PV of its market value at the end of Year 3
= $1,000 ÷ 1.08^3 = $793.83

Total PV = $13,017.58 + $793.83 = $13,811.41

NPV = $13,811.41 – $12,000 = $1,811.4

As the NPV of ***Brand B*** is higher, Mr Wong should buy ***Brand B***.

第 7 步 (i) 如果淨現值是正數，郭先生應該購買該電腦，因為這為他的公司帶來的財務利益大於成本。
(ii) 如果淨現值是負數，郭先生不應購買該電腦，因為這會為他的公司帶來財務損失。
(iii) 如果淨現值等於零，郭先生也應該購買該電腦，因為他所賺取的金額正好等於資本成本。
由於淨現值是**正數**，郭先生應該購買這台電腦。

例子 8

黃先生想為他的餐廳添置一台洗碗碟機。他正考慮兩個品牌:

✧ **品牌甲**的售價為 10,000 元，**品牌乙**的售價則為 12,000 元。

✧ 兩個品牌的洗碗碟機都可使用三年，在第三年年底可以 1,000 元賣出。

✧ 黃先生估計它們在未來三年年底可帶來的淨現金流入量:

	第一年	第二年	第三年
品牌甲	$5,000	$4,000	$3,000
品牌乙	$6,000	$5,000	$4,000

如黃先生的資本成本為 每年 8%，他應購買哪個品牌的洗碗碟機？

比較兩個品牌的洗碗碟機的淨現值:

品牌甲

每年淨現金流入量的總現值
= ($5,000 ÷ 1.08) + ($4,000 ÷ 1.08^2) + ($3,000 ÷ 1.08^3)
= $10,440.48

第三年年底洗碗碟機市值的現值
= $1,000 ÷ 1.08^3 = $793.83

總現值 = $10,440.48 + $793.83 = $11,234.31

淨現值 = $11,234.31 – $10,000 = $1,234.3

品牌乙

每年淨現金流入量的總現值
= ($6,000 ÷ 1.08) + ($5,000 ÷ 1.08^2) + ($4,000 ÷ 1.08^3)
= $13,017.58

第三年年底洗碗碟機市值的現值
= $1,000 ÷ 1.08^3 = $793.83

總現值= $13,017.58 + $793.83 = $13,811.41

淨現值= $13,811.41 – $12,000 = $1,811.4

由於**品牌乙**的淨現值較高，黃先生應購買**品牌乙**的洗碗碟機。

Nominal versus effective rate of return

Nominal rate of return

- ○ The nominal rate of return is the rate of interest **stated on** financial instruments.
- ○ It is also called the nominal interest rate or **quoted interest rate**.

Example 9

- ✧ Today, Mr Lee deposits \$100 in a bank for three years.
- ✧ The annual interest rate is **10%.**
- ✧ The interest is compounded annually.
- ✧ Nominal rate of return is **10%.**

Effective rate of return (ERR)

- ○ The effective rate of return reflects the effect of the **frequency** of compounding on the **actual** investment return.
  - ● The more **frequently** interest is compounded, the **larger** the sum at the end of the period because the interest grows **faster**.
- ○ It is also called the **effective annual rate (EAR)** or **effective interest rate**.
- ○ It is commonly used to **compare** different investment plans.

Effect of changing the frequency of compounding

Example 10

A bank offers a nominal annual interest rate of **10%.**

	1 Jan	31 Mar	30 Jun	30 Sept	31 Dec
Compounded once a year	100 → (100 × 1.1) → 110				
Compounded twice a year	100 → (100 × 1.05) → 105 → (105 × 1.05) → 110.25				
Compounded 4 times a year	100 → (100 × 1.025) → 102.5 → (102.5 × 1.025) → 105.06 → (105.06 × 1.025) → 107.69 → (107.69 × 1.025) → 110.38				

	Effective rate of return		
Compounded once a year	$\frac{\$110 - \$100}{\$100} \times 100\%$	= 10%	Same as the nominal rate
Compounded twice a year	$\frac{\$110.25 - \$100}{\$100} \times 100\%$	= 10.25%	Higher than the nominal rate
Compounded 4 times a year	$\frac{\$110.38 - \$100}{\$100} \times 100\%$	= 10.38%	Higher than the nominal rate

名義回報率和實際回報率

名義回報率

- ○ 名義回報率是金融工具上所**列明**的利率。
- ○ 又稱**名義利率**或**報價利率**。

例子 9

- ✧ 李先生在今天把 100 元存入銀行，為期三年。
- ✧ 銀行的年利率為 **10%**。
- ✧ 每年以複息計算利息。
- ✧ 名義利率是 **10%.**

實際回報率 (ERR)

- ○ 實際回報率能反映複息計算的**頻率**對**實際**投資回報的影響。
  - ● 複息計算的**頻率**越高，利息的增長**越快**，最終所得 的回報也就**越高**。
- ○ 又稱**實際年利率**或**實際利率**。
- ○ 實際回報率常用來**比較**不同的投資計劃。

改變複息計算頻率的影響

例子 10

假設銀行的名義回報率為每年 **10%**。

	1/1	31/3	30/6	30/9	31/12
每年以複息計算利息一次	100 → (100 × 1.1) → 110				
每年以複息計算利息兩次	100 → (100 × 1.05) → 105 → (105 × 1.05) → 110.25				
每年以複息計算利息四次	100 → (100x1.025) → 102.5 → (102.5x1.025) → 105.06 → (105.06x1.025) → 107.69 → (107.69x1.025) → 110.38				

	實際回報率		
每年以複息計算利息一次	$\frac{\$110 - \$100}{\$100} \times 100\%$	= 10%	與名義回報率相同
每年以複息計算利息兩次	$\frac{\$110.25 - \$100}{\$100} \times 100\%$	= 10.25%	比名義回報率高
每年以複息計算利息四次	$\frac{\$110.38 - \$100}{\$100} \times 100\%$	= 10.38%	比名義回報率高

Effect of changing the frequency of compounding

- The more frequently interest is compounded, the **larger** the final amount will be.
- Whenever the frequency of compounding differs, we should compare different investment plans based on their **effective rates of return** instead of their **nominal rates of return**.

Equation for calculating effective rate of return

$$\text{ERR} = \left(1 + \frac{i}{m}\right)^m - 1$$

Example 11

Nominal annual rate = **10%**; compounded **twice** a year:

$$\text{ERR} = \left(1 + \frac{10\%}{2}\right)^2 - 1 = 10.25\%$$

Nominal annual rate = **10%**; compounded **four** times a year:

$$\text{ERR} = \left(1 + \frac{10\%}{4}\right)^4 - 1 = 10.38\%$$

改變複息計算頻率的影響

- 複息計算的頻率越高，最終所得的金額便**越大**。
- 我們比較不同的投資計劃時，若計劃的複息計算頻率不同，我們便不應比較它們的**名義回報率**，而應比較它們的**實際回報率**。

計算實際回報率的公式

$$\text{ERR} = \left(1 + \frac{i}{m}\right)^m - 1$$

例子 11

名義年回報率 = **10%**；每年以複息計算利息**兩次**：

$$\text{ERR} = \left(1 + \frac{10\%}{2}\right)^2 - 1 = 10.25\%$$

名義年回報率 = **10%**；每年以複息計算利息**四次**：

$$\text{ERR} = \left(1 + \frac{10\%}{4}\right)^4 - 1 = 10.38\%$$